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# "Paradoxical" Production Prices in Single and Joint Production Techniques

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***Abstract:*** First, it shall be reminded when and for what reasons "paradoxes", that is negative, indefinite tending towards ±?, and zero production prices appear in single production techniques. Next, we present a separable production technique, that is a production technique, which is divided into at least two parts, one of which outweighs ordinaly not cardinaly to all the other parts with respect to labour productivity and to r- productivity, and we show that here too not only "paradoxical prices" appear, but also indefinite arithmetical values of the profit rate, as well as tending towards ±? nominal wage rates. In addition, w-r-relations with negative w and r, and positive slope, as well as positive profits with negative surplus product appear. Finally, negative "labour values" appear. All these paradoxes are a consequence of the implicit presupposition that productivity and r-productivity in the production of each commodity is in all the production processes of this commodity the same, which contradicts the separability of the technique. As a result, the neoricardian interpretation of profit, as well as Steedman's proof of negative " labour values" cannot be substantiated.

***Keywords:*** "Paradoxical prices" and w-r-relation in joint production, surplus approach, Negative "labour values".

***JEL Classifications:*** B51

## 1. The issue

In the following, we will refer to "paradoxes", that is negative, indefinite, tending towards $\pm\infty$, and zero prices, which appear in single production techniques (1st Section). In the 2nd Section, we will discuss the same "paradoxical" prices, as well as the "paradoxical " forms of the $w$-$r$-curve. We will also show that the surplus approach, that is the neoricardian interpretation of (positive) profit by (semi)positive surplus product, cannot be substantiated. In the 3rd Section, we will prove that negative "labour values", the existence of which was proved by Steedman, are inference of his implicit presupposition that in a separable technique the labour productivity in the production of each commodity is in all the production processes that produce it the same, which contradicts the fact that in this technique this is impossible, because the one part outweighs ordinaly to the other with respect of labour productivity. Therefore, neither negative values nor negative surplus value and even negative surplus value with positive profit exist, as Steadman and the Neoricardians claim. We conclude the article with some brief remarks on joint production techniques with a dominant part (Filippini and Fillipini, 1982), or, which is the same, on joint production separable techniques, and their concealment by the Neoricardians.

## 2. "Paradoxes" in Single Production Techniques

It is already known that in those single production reducible techniques, in which

(a) the non-basic commodities enter into their own production in a proportion greater than that in which the basic commodities enter into their own production; and

(b) the prices have been normalized with normalization commodity a commodity, which does not contain any non-basic commodity, non-positive and non-finite prices appear (Stamatis 2019). If $\overline{R}_1$ and $\overline{R}_{II}$ are the maximum profit rates of the basic and the non-basic part[2], then, according to the *w*-*r*-relation derived from the above-mentioned normalization, the resulting maximum profit rate $r_{(w=0)}$, $r_{(w=0)} = \overline{R}$ is equal to $\overline{R} = \overline{R}_I$. At the same time, for each *r*, $0 \leq r < \overline{R}_{II}$, the prices of the non-basic commodities are positive (those of the basic commodities are positive anyway due to the normalization), for $r = \overline{R}_{II}$ the same prices are indefinite, and for each *r*, $\overline{R}_{II} < r \leq \overline{R}_I$, they are negative. For r that tends from left (right) towards $\overline{R}_{II}$, the prices of the non-basic commodities tend towards $+\infty$ ($-\infty$).

If one normalizes the prices with a normalization commodity a commodity that consists only of non-basic commodities or contains at least one non-basic commodity, then the negative, the indefinite, and the prices that tend towards $\pm\infty$ disappear. However, in this case, for $w = 0$ and therefore $r = r_1 = \overline{R}_I$ and $r = r_2 = \overline{R}_{II}$, the prices of all the basic commodities are zero (Stamatis 2019).

Thus, the appearance of "paradoxical" prices in single production reducible techniques depends on the type of the normalization. This is because, as we know, the prices are the prices of the commodities of the normalization subsystem, which, using the given technique, produces as its net product the chosen normalization commodity (Stamatis 2019).

As is well known, in single production reducible or non-reducible techniques, the labour values are always unambiguously defined and positive.

## 3. "Paradoxes" in Joint Production Techniques

Negative prices appear in joint production already in irreducible techniques. This stems from the presupposition that for common for all the commodities *w* and *r*, there exists a single price for each commodity, the same, regardless of the production process in which its various quantities are produced, even though this is incompatible with positive prices of commodities for productive- technical reasons. These reasons lie in the fact that the minimum labour productivity may not be the same for every commodity in every production process (partial productivity), but may differ from process to process.

We call these joint production techniques separable, when they can be divided into two (at least) parts, each of which

(a) may, using only a few and not for all of the production processes, produce all the commodities produced by the technique; and

(b) its total labour productivity is ordinaly-*not cardinaly!*—comparable to that of the other part of the technique .

By minimum partial labour productivity of a commodity we mean the labour productivity in the production of this commodity in one of the production processes in which it is produced, provided that the production of the other commodities produced in this particular process does not cost any labour.

Next, we will discuss the form of the *w*-*r*-relation of a separable technique, as well as the dependence of prices on the profit rate for various normalizations of prices and the behavior of the latter in changes in the profit rate. We choose the so-called "chinese" way of addressing the issue. Just as the chinese language uses the sign ot for a particular signifier to denote an abstract one that includes this particular signifier, so we will address our issue here with the help of a specific numerical example of a separable technique.

Let us assume the technique $[B, A, \ell]$, where $B, B = \begin{pmatrix} 6 & 1 \\ 9 & 34 \end{pmatrix}$, is the *nxn* output matrix, $A, A = \begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix}$, is the matrix nxn of material inputs, $(B\text{-}A)$, $(B\text{-}A) = \begin{pmatrix} 6 & 1 \\ 9 & 34 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 9 & 24 \end{pmatrix}$, is the matrix of net outputs, and

$\ell, \ell = (1,1, \ldots,1)$ is the *1xn* vector of inputs in labour. This technique is obviously separable, because (a) each of the two processes can operate independently of the other and produce both, that is all the commodities produced by the technique; and (b) the second process is more productive than the first one, since the following holds:

$$\frac{1}{9} \leq \frac{1}{24},$$

where $(1\ ,9)^T$ and $(1,\ 24)^T$ are respectively the net products in commodities 1 and 2 per unit of labour of processes 1 and 2 respectively. As can be seen directly from the above the relation the minimum partial labour productivity in the production of commodity 1 is in both parts of the technique equal to one unit of commodity 1 per unit of labour, while the minimum partial labour productivity in the production of commodity 2 is in the first part equal to 9 and in the second part equal to 24 units of commodity 2 per unit of labour. Moreover, as can be seen directly from the above relation, the total productivities of the two parts are comparable to each other, that is they are not incomparable to each other[4], but, however, they are only *in ordinaly not cardinally* comparable to each other: It can be said that the one is greater than the other, but not how many times greater. This means that the two vectors of the above inequality are not collinear, that is proportional to each other.

For the prices $p_1$ and $p_2$ the following holds:

$$6p_1 + p_2 = 5p_1\,(1+r) + w \tag{1}$$

$$9p_1 + 34p_2 = 10p_1\,(1+r) + w. \tag{2}$$

Commodities 1 and 2 are both basic commodities. In single production all commodities are basic, when $(I\text{-}A)$ and, ultimately, $A$ is irreducible. Let us see how it goes in joint production. Since the outputs (inputs) are inputs (outputs) with a changed sign, a joint production technique $(B,\ A)$ can be converted to a single production technique by subtracting and adding to $B$ the $I$ and subtracting from $B$ and $A$ the $B$, so we get a technique with an output matrix $B - I + I - B(=I)$ and an input matrix $A–B$, that is the single production technique $[I, (A–B)]$. This technique is irreducible and therefore produces only basic commodities, when $[I,\ (A–B)]$ and, ultimately, $(B\text{-}A)$ is irreducible. The last matrix is in our numerical example, as seen above, strictly positive and therefore irreducible. Thus, commodities 1 and 2 are both basic commodities[5].

If we normalize the prices with normalization commodity the commodity 1, setting the price of one unit of it equal to one unit of fictitious money per unit of commodity 1

$$p_1 = 1, \tag{3}$$

then from (1), (2), and (3) we get for the $w$-$r$-relation and the $p_2$-$r$-relation respectively

$$w = \frac{r^2 - 2.6r + 0.3}{0{,}46 - 0.2r} \tag{4}$$

$$p_2 = -\frac{5r+8}{0.23 - 10r}. \tag{4α}$$

For $w = 0$ we get from (4)

$$R_1 = 0.121$$

and

$$R_2 = 2.478.$$

The following Diagrams I and II illustrate $p_2$ and the nominal wage rate $w$ as functions of the profit rate $r$.

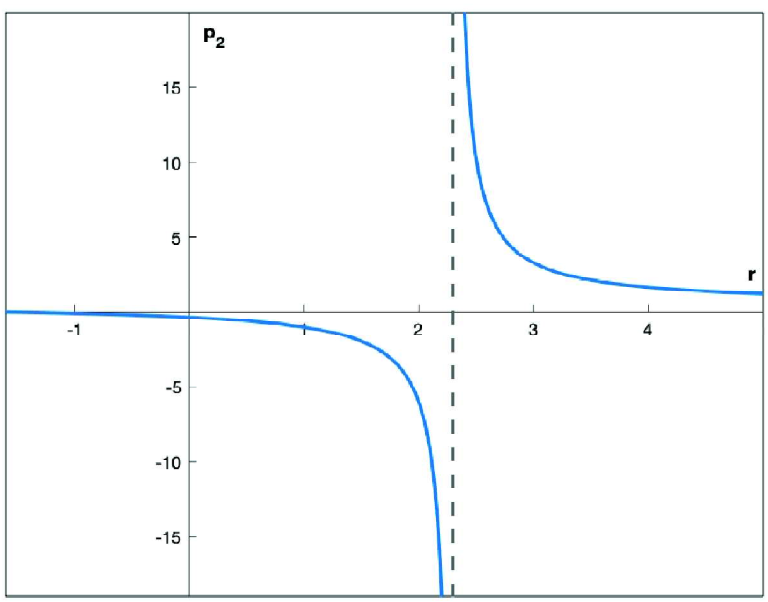


**Diagram I**

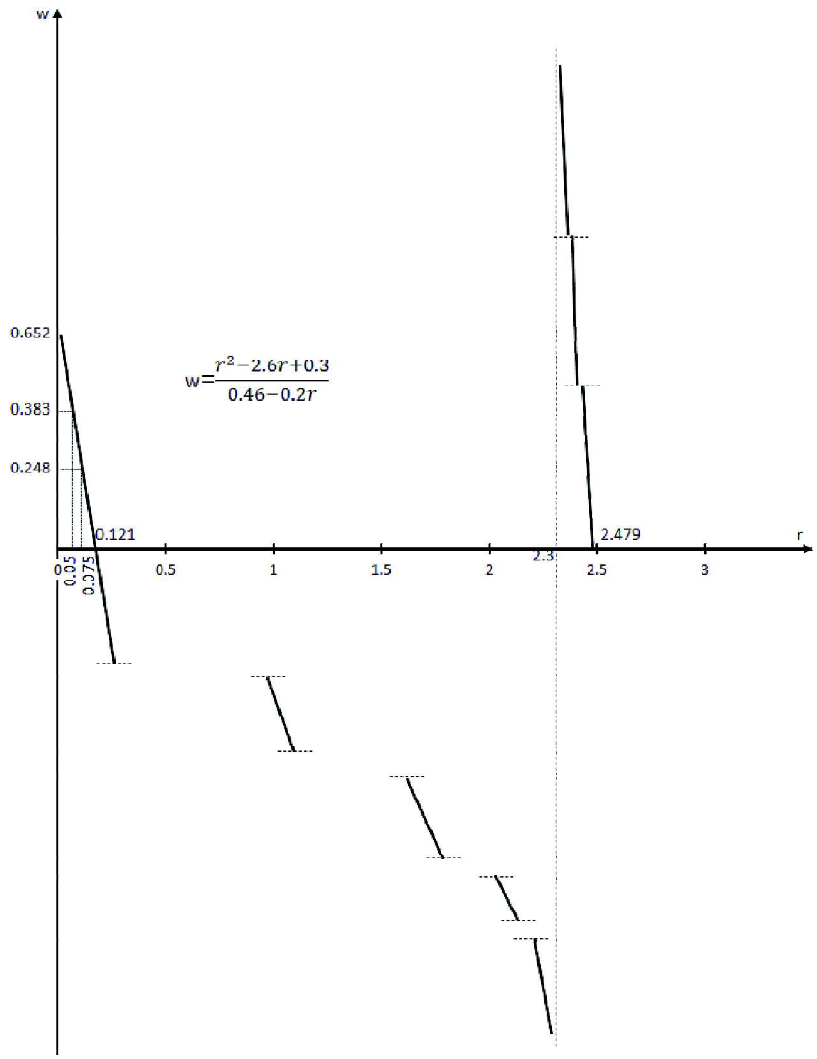


**Diagram II**

Here, as in the case of single production reducible techniques, all the "paradoxes" concerning the prices of the commodities appear. The reason for the appearance of these "paradoxical" prices is the one that has already been mentioned: The petition that a uniform profit rate exists, even though this is impossible due to the fact that the "productivities of capital" in the two parts in each case given reducible or separable technique are different. With regard to separable techniques, an additional reason is added: The petition that labour productivity *should be the same for each commodity in all production processes,* whether this is the case or, as in our case, is not the case. In the given case, in which this does not happen, phenomena arise that do not appear in single production reducible techniques of the types that we discussed earlier. We will refer to them below.

Concerning *w-r*-curve in Diagram II it shows a "paradoxical " form: It consists of two branches. The first branch starts at the point ($w$=0.652/$r$=0), it is positive, convexly decreasing, it intersects the $r$ axis at the point ($w$=0/$r$ = $R_1$=0.121), it becomes negative and tends, for $r \rightarrow 2.3$, towards $-\infty$. The part of this branch located in the positive quadrant appears to be a normal *w-r*-curve. But it is not. Because at this "normal" part negative prices of commodities appear.

As a consequence of the appearance of negative prices, there are cases, for example, where the profit and the nominal capital are positive magnitudes (the profit rate is also positive), while the surplus product is a (semi) negative magnitude. This is the case where the real wage rate is $(1, 1.25)^T$. In this case, where $p_1 = 1$, $p_2 = -0.354$, $w = 0.556$, and $r = 0.017$, the surplus product of process 1 is semi-negative and equal to $(0, - 0.25)^T$, the profits (0+0,2.5·0.354) are (such as the nominal capital) positive. Exactly the same happens, as one can easily come to the conclusion that, when the real wage rate is equal to $(1, 2)^T$, the surplus product of process 1 is equal to (0, - 1), the lowest of the two nominal wage rates is equal to 0.246, that is lower than 0.652 (resulting for $r$=0). Here again, in process 1 the positive profits $(p_1, p_2)$ $(0, -1)[=(1, -0.376)$ $(0, - 1)^T$ = +0.376] coexist with the (semi) negative surplus product $(0, -1)^T$. So we have -in contrast to the neoricardian theory of profit, according to which the source of profit is the surplus product- the coexistence of positive profit and (semi)negative surplus product.

The, so to speak, most scandalous consequence of the appearance of negative prices in the interval of $r$, $0 \leq r \leq 0.121$ is the following: It is possible for the nominal wage rate to decrease (increase) with an increasing (decreasing) real wage rate. Thus, when the real wage rate is equal to $(0.75, 1)^T$, the nominal wage rate is +0.383. However, when the real wage rate increases to $(1, 2)^T$, the *nominal wage rate decreases* to +0.246. Or, when the real wage rate increases from $(1 , 1.25)^T$ to $(1, 2)^T$, the *nominal* wage rate decreases from 0.556 to 0.246! Finally, as we have seen, while in single production reducible techniques, such as the above, the prices -through the appropriate normalization or arbitrary restriction of the profit rate by the lowest maximum profit rate- are positive, in joint production separable techniques in this interval of $r$ negative prices appear. The concealment of the issue of negative prices, the issues associated with it, as well as other problems of the provenance that we just mentioned is the usual way in which Neoricardians address it. For $r$, $0.121 < r < 2{,}3$, the nominal wage rate becomes negative and tends towards $-\infty$.

The second branch starts at the point ($w=0/r=R_2=2.478$) and tends increasing, for $r \to 2{,}3$, towards $+\infty$. Here, there are also negative prices of commodities. Of course, it is not possible for this branch to take the place of a *w*-*r*-curve, since, apart from anything else, here, when $r$ tends decreasing towards 2.3, w tends towards $+\infty$, that is the maximum $w$ does not correspond to zero $r$. Or should, however, this second branch represent the real *w*-*r*-curve, according to the Neoricardians, since this branch, and not the first one, gives for a given $w$ the greatest $r$?

If we normalize the prices with

$$p_2 = 1, \tag{5}$$

then for $p_1$ we get

$$p_1 = \frac{10r-23}{5r-8} \tag{6}$$

and the new *w*-*r*-curve is a parabola:

$$w = \frac{50(r-0.121)(r-2.478)}{-(5r+8)} \tag{7}$$

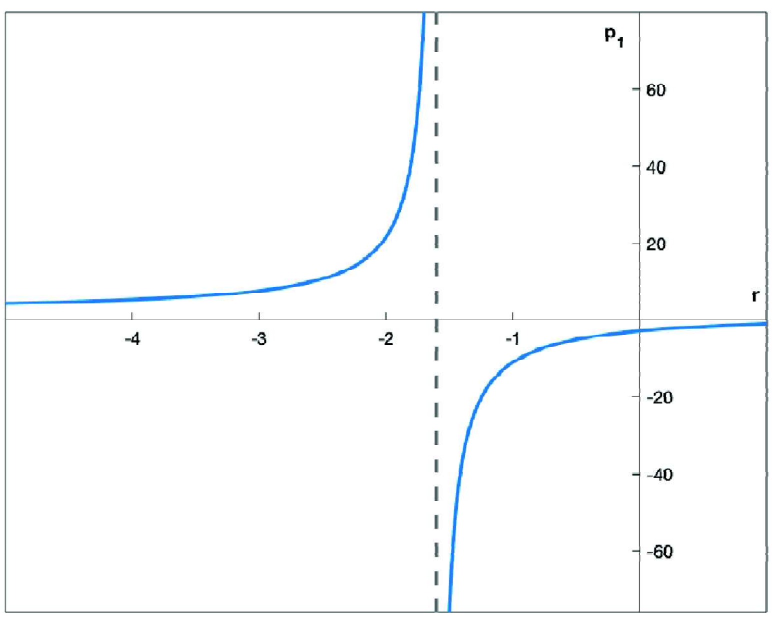


**Diagram III**

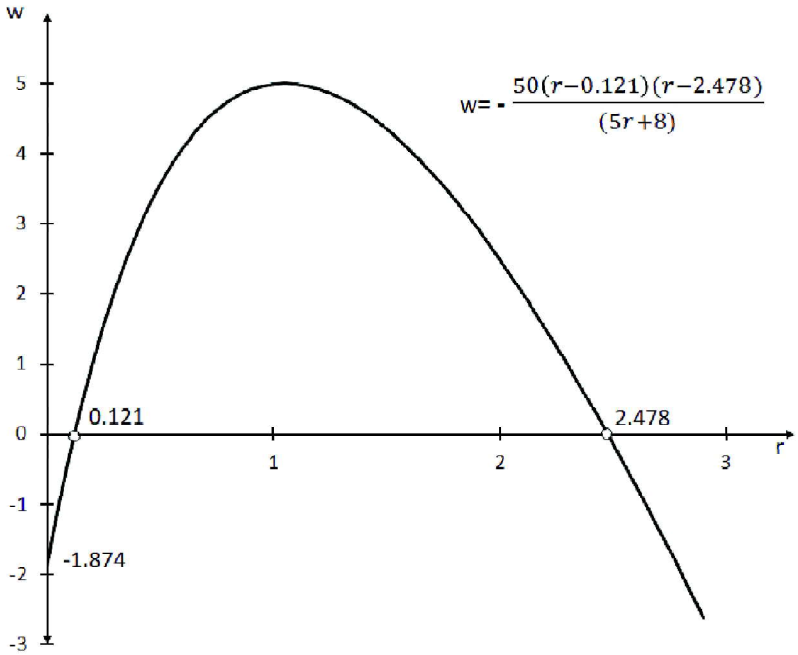


**Diagram IV**

Diagrams III and IV represent $p_1$ and $w$ as functions of $r$. The $w$-$r$-curve starts at the point ($w = -1.87/r=0$), it is increasing until it intersects the $r$-axis at the point ($w = 0/r = R_1 = 0.121$). Its cathodic branch intersects the r-axis at the point ($w = 0/r = R_2 = 2.478$). In the important for the Neoricardians interval $0 \leq r \leq R_1$ (=0.121), the nominal wage rate $w$ is *negative* and *increases* (*decreases*) with an increasing (decreasing) profit rate $r$. That is the $w$-$r$-relation in this interval *is not an inverse relation*! The cathodic branch of the parabola cannot be considered to represent the $w$-$r$-curve, because its starting point, that is the maximum nominal wage rate, does not correspond to a zero, but to a positive profit rate! Or should, however, this second branch represent the real $w$-$r$-curve, according to the Neoricardians, since this branch, and not the first one, presents a negative slope and gives for a given w the greatest $r$?

In the given case of separable techniques, the negative, the indefinite, and the prices that tend towards $+\infty$ or $-\infty$ can by no means be avoided. Nor, as we will see below, by the Sraffian normalization of prices. In our example, one can realize that there are two Sraffian standard systems: one with the standard ratio $R_1$, and the standard net product $q_1$

$$q_1 = (0.136 - 0.120)^T$$

and a second with the standard ratio $R_{II}$,

$$R_{II} = 2.478$$

and the net standard product $q_2$

$$q_2 = (7.766 \quad 19.667)^T.$$

Sraffa finds nothing absurd in a standard net product that also contains negative quantities of commodities, such as the first of the two given here:

"The raison d'être of the Standard system, however, is to provide a Standard commodity. And in the case of the latter is fortunately no insuperable difficulty in conceiving as real the negative quantities that are liable to occur among its components. These can be interpreted, by analogy will the accounting concept, as liabilities or debts, while the positive components will be regarded as assets.Thus a Standard commodity, which includes both positive and negative quantities can be adopted as money of account without too great a et retch of the imagination provided that the unit is conceived as representing, like a share in a company, a fraction of each asset and of each liability, the latter in the shape of an obligation to deliver without payment certain quantities of particular commodities." (Sraffa, 1960: § 56).

Sraffa considers that irregularities, such as prices equal to infinity, which may occur for normalizations of prices with normalization commodity a standard net product with standard ratio (=maximum profit rate) greater than that with the minimum standard ratio, can be avoided, if we normalize the prices with a normalization commodity the standard net product with the lowest standard ratio. And he continues: "This is the only Standard product in terms of which, at all the levels of the wage from 1 to 0 (and so at all the levels of the rate of profit from 0 to its maximum), it is possible for the prices of commodities to be finite." (Sraffa, 1960: § 64).

In addition, Sraffa considers that if one wants to avoid the appearance of irregularities, such as the equal to infinity resulting prices, when one normalizes the prices with normalization commodity one of the standard net products that are greater than that with the minimum standard ratio (the minimum maximum profit), he must normalize the prices with normalization commodity the standard net product of the standard system with the lowest standard ratio, in our case, with the first one, which also contains a negative quantity of commodity. "This is the only Standard product in terms of which, at all the levels of the wage from 1 to 0 (and so at all the levels of the rate of profit from 0 to its maximum), it is possible for the price of commodities to be finite." (Sraffa, 1960: § 64).

We will normalize the prices successively with the two given here standard net commodities by

$$7.766p_1 + 19.660p_2 = 1 \tag{8}$$

and

$$0.136p_1 - 0.120p_2 = 1 \tag{8?}$$

and we will express $w$, $p_1$, and $p_2$ as functions of $r$. In the meantime, we will check the mentioned positions of Sraffa.

From (1), (2), and (8) we get

$$W = \frac{50r^2 + 130r - 15}{175.99r - 21.282} = \frac{r^2 + 2.6r - 0,3}{3.52r - 0.426} = -\frac{(t - 0.121)(r - 2.478)}{3.52(r - 0.121)} = \frac{2.478}{3.52} - \frac{1}{3.52}r \Rightarrow$$

$$w = 0.705 - \frac{1}{R_{II}}r \tag{9}$$

where $R_{II} = 2.478$ and $w = w_{max} = 0.705$.

$$p_1 = \frac{r - 2.3}{17.6r - 2.130} = \frac{r - 2.3}{17.6(r - 0.121)} \tag{10}$$

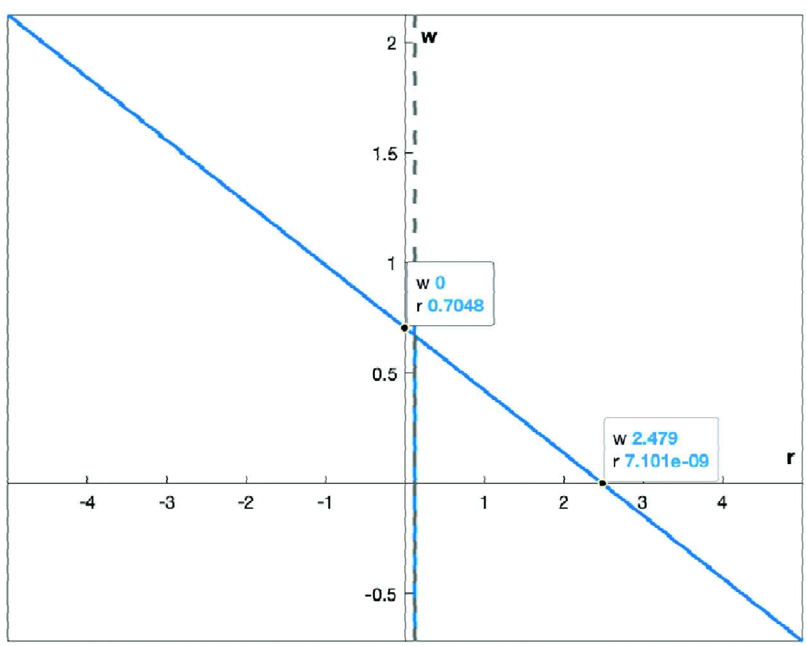


**Diagram V**

$$p_2 = \frac{r + 1.6}{35.2r - 4.26} = \frac{r + 1.6}{35.2(r - 0.121)} \tag{11}$$

The following Diagrams V, VI, and VII show the relations (9), (10), and (11).

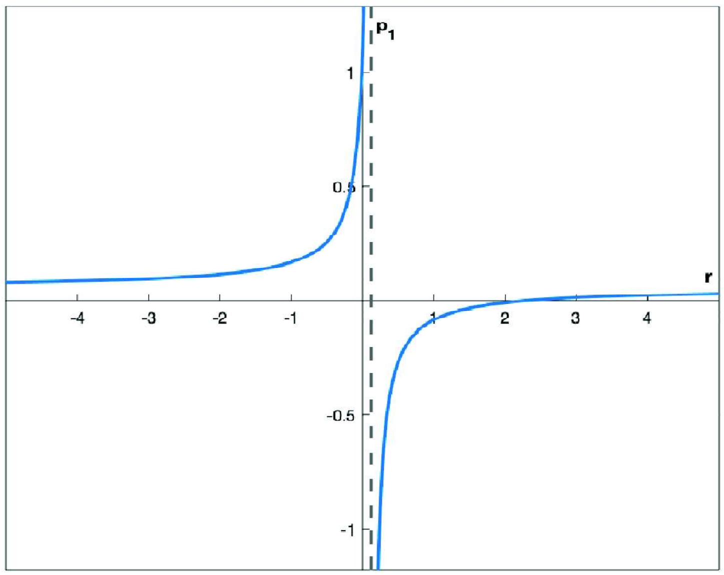


**Diagram VI**

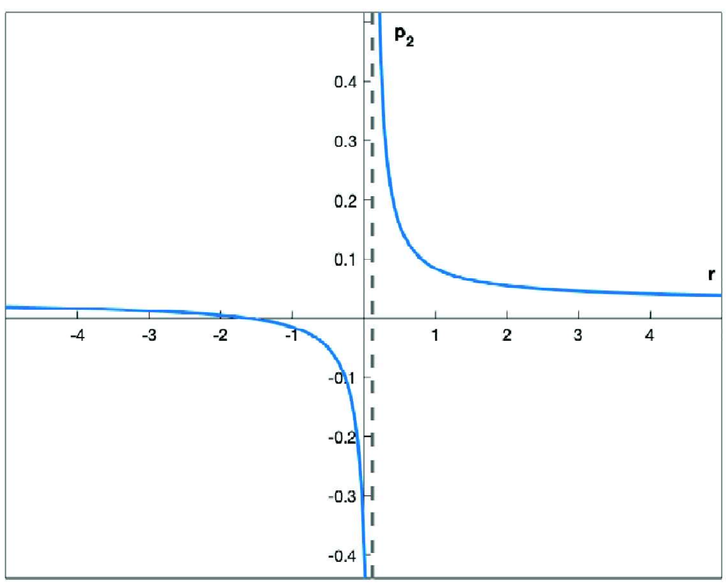


**Diagram VII**

The price $p_1$ is positive for $r = 0$ and indefinite for $r$, $0 < r < 0.121$. For $r > 0.121$ it is initially negative, it becomes positive and increases with increasing $r$. For $r$, $0 \leq r \leq 0.121$, the price $p_2$ is indefinite and, for $r$ tending from right towards $r$, $r = 0.121$, it tends towards $\pm\infty$.

So not only prices that tend towards $\pm\infty$ appear, in contradiction to what Sraffa claims, but also indefinite, as well as negative prices, when one normalizes the latter with a normalization commodity and a standard net product with a standard ratio greater than that with the minimum, as is the case here by (8).

If we normalize the prices with a normalization commodity the standard net product with the negative quantity of commodity 2 by

$$0.136p_1 - 0.12p_2 = 1, \tag{8α}$$

then we get for $w$, $p_1$, and $p_2$ as a function of $r$

$$w = \frac{-6.8r^2 + 17.68r - 2.04}{0.136(0.7r - 4.088)} \tag{13}$$

$$p_1 = \frac{1.36r - 3.128}{0.136(0.76r - 4.088)} \tag{14}$$

and

$$p_2 = \frac{5r - 8}{0.76r - 4.088} \tag{15}$$

The following Diagrams VIII, IX, and X illustrate the relations (13), (14), and (15).

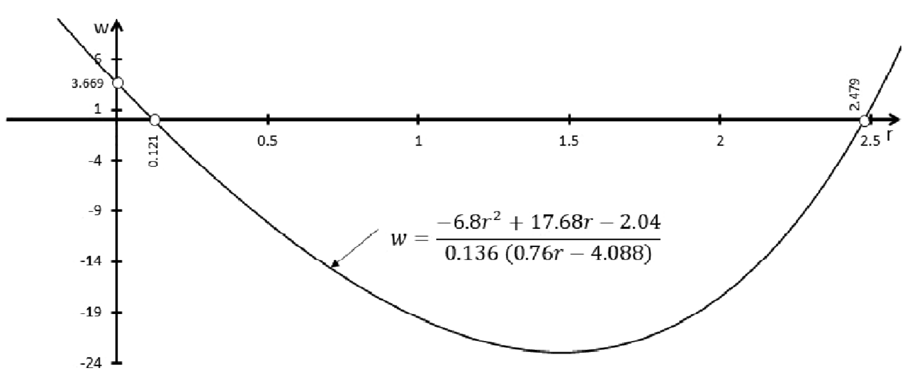


**Diagram VIII**

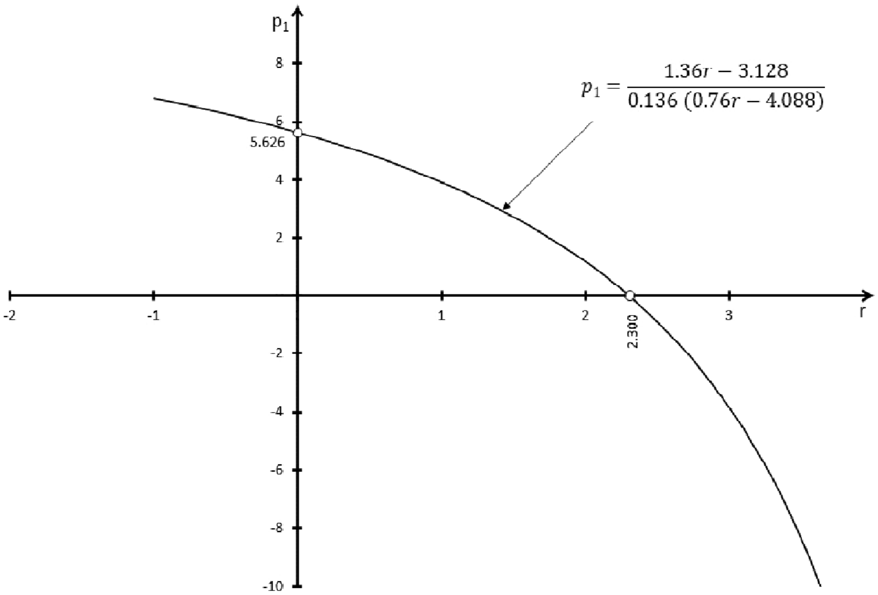


**Diagram IX**

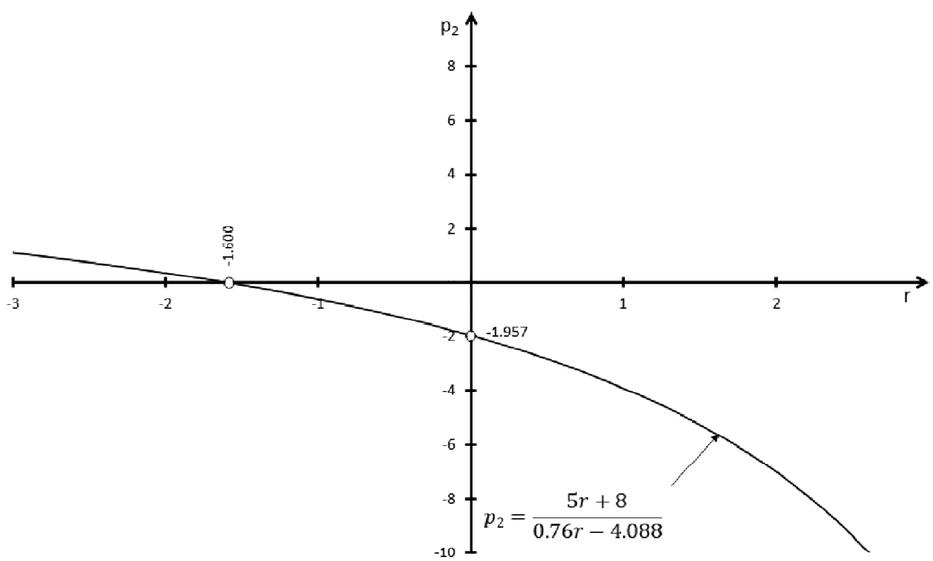


**Diagram X**

The *w*-*r*-curve, the curve of Diagram VIII, is not linear in the interval of *r*, $0 \leq r \leq 0.121$, despite Sraffa's claim. The maximum profit rate *R* is equal to 0.121 and the maximum wage rate $w_{(r=0)}$ is equal to 3.669. It appears from the diagrams that, in the above economically significant interval of $r$, $p_1$ is positive and $p_2$ is negative. Anyone who wishes can explore for their own leisure how negative prices contribute to a uniform profit rate in that case, in which the net standard product commodity functions as normalization commodity includes also negative commodity quantities, that is in the case where the production system produces a net product of the same composition with its means of production, which also contains negative quantities of commodities, as well as because, while the normalization commodity is a Sraffian standard net product, the $w$-$r$-relation is not linear, but convex. Thus, the Sraffian normalization by (8a) appears to be a usual normalization, the normalization commodity of which also contains commodity 2. In addition: With this normalization, the prices that tend towards $\pm\infty$ (and the indefinite prices) disappear, but negative prices of commodities appear instead.

Neither does the normalization of prices with a normalization commodity the Sraffian standard commodity solve any of the problems we have pointed out.

## 4. The Labour Values in Joint Production Techniques

We already know from Sraffa that the prices $p$ obtained for $r = 0$ are equal to the "labour values" $\omega$. By normalizing with $w = 1$ and setting $r = 0$, we get from (1) and (2)

$$6p_1 + p_2 = 5p_2 + 1. \quad (1\alpha)$$

$$9p_1 + 34p_2 = 10p_2 + 1. \quad (2?)$$

From (1α) and (2α) we get

$$p_1 = \omega_1 = 23/15 \text{ and } p_2 = \omega_2 = -8/15.$$

To our knowledge, the minimum labour productivity with respect to commodity 2 is in the first part of the technique equal to 9 units of commodity 2 per unit of direct labour[6]. And that the minimum labour productivity with respect to the commodity itself is in the second part of the technique equal to 24 units of commodity 2 per unit of direct labour. These productivities are unequal as physical productivities and equal as nominal productivities, only when the price $p_2$ of commodity 2 is negative. So, according to the above we have

$$\omega_1 = 23/15 \quad (12)$$

and

$$\omega_2 = -8/15. \quad (13)$$

Equation (13) is, of course, a contradictio in objecto. Because it means that the production of a labour product, consisting of commodity 2, does not cost, but creates labour instead. Equation (13) does not hold, of course, because the production of commodity 2 does not cost, but creates labour, but it holds, as mentioned above, as a consequence of the unenforceable requirement that the productivity in the production of commodity 2 should be the same in both parts of the technique.

Equation (12) means two things: (a) that the minimum labour productivity in the production of commodity 1 is the same in both parts of the technique; and (b) that all direct labour that enters both into the first and the second part of the technique for the production of the net product, which is equal to the unit, enters exclusively into the production of commodity 1, since, as we have seen, due to $\omega_2 = -8/15$, in the production of commodity 2 in each of the two parts of the technique not only is does not cost, but creates labour instead. But not only that. And the quantity of labour 8/15 created in the second part also enters into the production of commodity 1, so that in the latter enters the quantity of labour in total.

A thought experiment may make the above perfectly clear. It shall be reminded that labour productivity in the production of a commodity is the inverse of the value of the latter. We start from the minimum labour productivities in the production of commodity 1 in processes 1 and 2, which are equal to 1 unit of commodity 1 per unit of direct labour in process 1 and 1 unit of commodity 1 per unit of direct labour in process 2, respectively. This means that in process 1 in the production of 9 units of commodity 2 and in the production of 24 units of the same commodity in process 2 no labour is entered and therefore the labour productivity in the production of these commodities in processes 1 and 2 respectively is equal to 9 units of commodity 2 and 24 units of the same commodity per zero or infinitesimal quantity of labour, that is infinitely large.

Consequently, the minimum labour productivities in the production of commodity 1 in processes 1 and 2 are respectively

$$\frac{1}{\omega_{11}} = 1$$

and

$$\frac{1}{\omega_{12}} = 1 \cdot$$

The corresponding maximum labour productivities in the production of commodity 2 in the same processes tend towards $+\infty$:

$$\frac{1}{\omega_{21}} = \frac{1}{\omega_{22}} \rightarrow +\infty,$$

where $\omega_{ij}$ is the value of commodity *i* in process *j*, *i*, *j* = 1, 2.

Apparently, there is no common labour value for commodity 1, while for commodity 2 there is, but it lacks economic sense.

Let us see how we could achieve a common labour value for all the processes of each of the two commodities. We first subtract labour from the production of commodity 1 and introduce it into the production of commodity 2, allocating it into processes 1 and 2 in the production of this commodity so that

$$\frac{1}{\omega_{11}} > 1,$$

$$\frac{1}{\omega_{12}} > 1$$

and

$$\frac{1}{\omega_{21}} = \frac{1}{\omega_{22}} \left( = \frac{1}{\omega_2} \right),$$

where $1/\omega_2$ is the weighted average labour productivity in the production of commodity 2.

At the same time, we require the following to hold

$$\frac{1}{\omega_{11}} = \frac{1}{\omega_{12}} \left( = \frac{1}{\omega_1} \right),$$

where $1/\omega_1$ is the weighted average labour productivity in the production of commodity 1.

The last two equations, however, contradict the fact that the production process 2 is, with regard to both commodities, more productive than the production process 1. This fact, of course, is contradicted by the following equations, which result from the last two, in particular the equations

$$\omega_{11} = \omega_{12} \ (= \omega_1)$$

and

$$\omega_{21} = \omega_{22}\ (=\omega_2).$$

We therefore reduce the labour that enters into both processes in the production of commodity 1, but, however, so that the first of the two above equations always holds and in order for the second equation to also hold.

From (1a) and (2a) one can easily come to the conclusion that, during the gradual transfer up to the transfer of all labour to the production of commodity 2, in which case holds

$$\omega_{11} = \omega_{12}\ (= \omega_1) = 0,$$

it is

$$\omega_{21} \neq \omega_{22}\ (\neq\omega_2).$$

The only way left to achieve the presupposed validity of the two equations

$$\omega_{11} = \omega_{12}\ \ (= \omega_1)$$

and

$$\omega_{21} = \omega_{22}\ (= \omega_2)$$

is, starting from its initial validity

$$\omega_{11} = \omega_{12}\ (= \omega_1) = 0,$$

to gradually transfer labour from the production of commodity 2 to that of commodity 1, so that the equation

$$\omega_{21} = \omega_{22}\ (= \omega_2)$$

is continues to be valid and in order for the following equation to hold

$$\omega_{11} = \omega_{12}\ (= \omega_1).$$

Of course, this means that $\omega_2$ becomes negative, while for a certain negative arithmetical value of $\omega_2$ the last equation holds, where $\omega_1$ is positive. These arithmetical values of $\omega_1$ and $\omega_2$ are, as follows from (1a) and (2a) for $p_1 = \omega_1$ and $p_2 = \omega_2$:

$$\omega_1 = 23/15 \text{ and } \omega_2 = -8/15,$$

where

$$\omega_1 + \omega_2 = (23/15) - (8/15) = 1.$$

The last equation represents the net value produced in process 1. The same amount of net value is produced in process 2:

$$9 \cdot [(23/15) + 24(-8/15)] = \frac{207}{15} - \frac{192}{15} = 1.$$

May I remind you that the net value produced in a process is equal to the direct labour that enters into it.

Let me say that Steedman's famous negative " labour values" are the result of the kind of the numerical example of the technique he has chosen. His example constitutes a joint production separable technique, one part of which presents a greater productivity of labour (Steedman 1975). The example is the following:

$$B = \begin{bmatrix} 6 & 3 \\ 1 & 12 \end{bmatrix}, \quad A = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}, \quad (B - A) = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}, \qquad \ell = (1,1).$$

The " labour values" we get, according to Steedman, are:

$$\omega_1 + \omega = 1$$

$$3\omega_1 + 2\omega_2 = 1$$

and therefore

$$\omega_1 = -1$$

and

$$\omega_2 = 2.$$

Steedman *et al.* base their claim on the invalidity of the Marxian theory of value and surplus value on the appearance of these negative " labour values" and a possible negative "surplus value" derived from them and associated with a positive profit.

## 5. On Separable Joint Production Techniques or on Joint Production Techniques with Dominance

We developed and used the concept of joint production separable techniques in a number of our studies (Stamatis 1979a and Stamatis 1983), to disprove the negative "labour values" and negative "surplus value" with positive profit. Shortly afterwards, Filippini and Filippini (1982) presented a short, elegant mathematical formulation of the same concept extended. The extension consists in the fact that the dominance of one over the other part of the technique concerns not only labour productivity, but also r-productivity, the profit per unit of labour, it holds, that is not only for zero, but also for a positive profit rate.

The first case concerns the negative "labour values" of Steedman, to which Filippini and Filippini explicitly refer as a consequence of the "dominance" of one part of the technique over the other in terms of labour productivity and Steedman's misrecognition. The second case concerns the negative prices in separable techniques. Neoricardians did not consider it necessary to draw conclusions about either their erroneous critique of the Marxian theory of labour value and surplus value or the lack of logical coherence of their price theory.

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